Lecture 5: Beyond the Standard Model

Neutrino masses, naturalness, and grand unification

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Introductory Lectures on the Standard Model

Purpose of Lecture 5

Central question

If the Standard Model works so well, why do we believe it is not the final theory?

In the previous lectures we built the Standard Model as

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

with spontaneous electroweak symmetry breaking.

Today we follow the BSM pattern:

$$\boxed{\text{neutrino masses} \;\Rightarrow\; \text{hierarchy problem} \;\Rightarrow\; \text{grand unification}}$$

with some phenomenological perspective on precision tests, flavour, and open questions.

Roadmap of the lecture

1. The SM as a successful but incomplete theory
2. Effective field theory viewpoint (sorry, thats my favourite!)
3. Neutrino masses and the Weinberg operator
4. Right-handed neutrinos and the seesaw mechanism
5. The gauge hierarchy problem
6. Grand unification and charge quantization
7. Final synthesis of the five lectures
8. Three tutorial problems

Boundary

We keep the discussion conceptual and calculational at the level of simple estimates.

What we have achieved

The Standard Model describes known non-gravitational particle physics by

$$\boxed{\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{fermion}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Yukawa}}.}$$

Its structure is fixed by

$$\boxed{SU(3)_C \times SU(2)_L \times U(1)_Y}$$

and by the representation assignments of quarks, leptons, and the Higgs field.

It accounts for

- QED and QCD interactions,
- parity-violating weak interactions,
- massive weak bosons and a massless photon,
- fermion masses and mixing,
- the observed Higgs boson.

Working perspective: precision as a microscope

The SM is not only a qualitative framework: it is tested quantitatively.

Typical precision observables include

$$M_W, \qquad \Gamma_Z, \qquad \sin^2\theta_{\mathrm{eff}}, \qquad Z \to f\bar{f},$$

flavour data, CP violation, and neutrino oscillations.

Message

Precision measurements probe the quantum structure of the SM.

$\boxed{\text{The SM is a triumph precisely because its radiative corrections work.}}$

But the SM is not the final answer

The SM leaves several questions unanswered:

- Why are neutrinos massive?
- What is dark matter?
- Why is there more matter than antimatter?
- Why is the electroweak scale so small compared with very high scales?
- Why do gauge charges look quantized?
- Why three generations?
- How is gravity included?

The SM is a complete low-energy theory, but not a complete theory of Nature.

The modern EFT viewpoint

Our Lecture 4 prepared the logic used in here.

If new physics lives at a high scale Λ, then at low energies we write

$$\boxed{\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots .}$$

Higher-dimensional operators are suppressed by powers of E/Λ.

Key idea

The SM is the leading term in an expansion in powers of energy over the new-physics scale.

Why the first correction matters

The leading correction beyond the renormalizable SM is dimension five:

$$\boxed{\mathcal{L}_5 \sim \frac{1}{\Lambda} Q^{(5)}.}$$

There is a remarkable fact:

Weinberg's observation

With the SM field content and gauge symmetry, there is a unique dimension-five operator.

It gives neutrino masses.

$\boxed{\text{Neutrino masses are the first expected sign of physics beyond the SM.}}$

Neutrino oscillations: the experimental clue

Neutrino oscillations imply that neutrino flavour states are not mass eigenstates:

$$\boxed{|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle.}$$

For two flavours,

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right).$$

Thus oscillations require

$$\boxed{\Delta m^2 \neq 0.}$$

Conclusion

At least two neutrinos are massive, so the minimal SM is incomplete.

Why minimal SM neutrinos are massless

In the minimal SM field content there is no right-handed neutrino:

$$\nu_R \quad \text{is absent.}$$

Therefore one cannot write the Dirac Yukawa term

$$\bar{L}\,\widetilde{\Phi}\,\nu_R\ ,$$

where $\widetilde{\Phi} = i\tau_2\Phi^*$ and Φ the Higgs $SU(2)_L$-doublet.

One also cannot write a renormalizable Majorana mass for ν_L, because ν_L belongs to the weak doublet

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$

$$\boxed{\text{Minimal SM gauge invariance and field content imply } m_\nu = 0.}$$

The Weinberg operator

The unique dimension-five operator can be written schematically as

$$\boxed{\mathcal{L}_5 \supset -\frac{c_{ij}}{\Lambda}\left(L_i^T C\widetilde{\Phi}^*\right)\left(\widetilde{\Phi}^\dagger L_j\right) + \mathrm{h.c.}.}$$

Here

$$\widetilde{\Phi} = i\tau^2\Phi^*.$$

After electroweak symmetry breaking,

$$\Phi \to \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix},$$

this becomes a Majorana mass term:

$$\boxed{(m_\nu)_{ij} \sim \frac{c_{ij}v^2}{\Lambda}.}$$

A scale estimate

Take

$$m_\nu \sim 0.05 \text{ eV}, \qquad v = 246 \text{ GeV}.$$

If $c \sim 1$, then

$$\Lambda \sim \frac{v^2}{m_\nu}.$$

Using

$$1 \text{ eV} = 10^{-9} \text{ GeV},$$

we get

$$\boxed{\Lambda \sim \frac{(246 \text{ GeV})^2}{5 \times 10^{-11} \text{ GeV}} \sim 10^{15} \text{ GeV}.}$$

The lesson

Tiny neutrino masses may be a low-energy window onto a very high scale.

Majorana versus Dirac neutrinos

Dirac mass

Requires ν_R:

$$\mathcal{L}_Y^\nu = -y_\nu \bar{L}\widetilde{\Phi}\nu_R + \mathrm{h.c.}$$

$$m_D = \frac{y_\nu v}{\sqrt{2}}.$$

Lepton number can be conserved.

Majorana mass

Particle and antiparticle are not distinct:

$$\mathcal{L}_M = -\frac{1}{2} M \nu_R^T C \nu_R + \mathrm{h.c.}$$

Lepton number is violated.

Neutrino masses ask whether lepton number is fundamental or accidental.

Right-handed neutrino and the seesaw

Add a gauge-singlet right-handed neutrino:

$$\nu_R \sim (1, 1, 0).$$

The most general renormalizable terms include

$$\mathcal{L} \supset -y_\nu \bar{L}\widetilde{\Phi}\nu_R - \frac{1}{2}M\nu_R^T C\nu_R + \mathrm{h.c.}.$$

After electroweak symmetry breaking,

$$m_D = \frac{y_\nu v}{\sqrt{2}}.$$

For $M \gg m_D$, the light neutrino mass is

$$\boxed{m_\nu \sim \frac{m_D^2}{M}.}$$

Seesaw idea

A very heavy sterile neutrino naturally produces a very light active neutrino.

Neutrino masses: what students should remember

1. Minimal SM neutrinos are massless.
2. Oscillations show that neutrinos are massive.
3. The lowest-dimensional SM-gauge-invariant correction is the Weinberg operator.
4. The estimate $m_\nu \sim v^2/\Lambda$ points to $\Lambda \sim 10^{14} - 10^{15}\,\mathrm{GeV}$ for order-one coefficients.
5. A simple ultraviolet completion is the seesaw mechanism.

Neutrino mass is the clearest established evidence for BSM physics.

What is the hierarchy problem?

The Higgs potential contains the mass parameter

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2.$$

(who has chosen the sign in front $\mu^2 > 0$ a minus is a mistery to me!)

The electroweak scale is set by

$$v^2 = \frac{\mu^2}{\lambda}, \qquad v \simeq 246 \text{ GeV}.$$

But we have reasons to suspect much higher scales:

$$\Lambda_\nu \sim 10^{14} - 10^{15} \text{ GeV}, \qquad M_{\rm Pl} \sim 10^{19} \text{ GeV}.$$

Question

Why is the Higgs/electroweak scale so much smaller than these high scales?

Quadratic sensitivity: the basic issue

Scalar masses are sensitive to heavy physics.

Schematically,

$$\boxed{\delta m_h^2 \sim \frac{\kappa}{16\pi^2}\Lambda^2.}$$

If Λ is very large, the correction is much larger than

$$m_h^2 \sim (125\ \mathrm{GeV})^2.$$

A delicate cancellation would be needed between the bare mass and loop corrections.

$\boxed{\text{This is the naturalness or gauge hierarchy problem.}}$

Why fermions and gauge bosons are different

The hierarchy problem is especially sharp for scalars.

- A massless fermion can be protected by chiral symmetry.
- A massless gauge boson can be protected by gauge symmetry.
- A light fundamental scalar is not protected in the same way.

EFT's viewpoint

In the EFT perspective, low-energy mass parameters should know about the high-energy theory.

$\boxed{\text{Why does the Higgs remain light?}}$

Possible solutions

Many proposed solutions introduce new dynamics near the TeV scale.

- Supersymmetry: boson and fermion loops cancel.
- Composite Higgs: the Higgs is not elementary.
- Extra dimensions: the meaning of high scales changes.
- Strong dynamics: electroweak symmetry breaking is dynamical.

2026 Status in the spirit of the lecture

These are beautiful ideas, but no definitive new particles have yet been established.

Naturalness remains a major guide, but not yet a discovery.

Motivation for grand unification

The SM gauge group is a product:

$$SU(3)_C \times SU(2)_L \times U(1)_Y.$$

This raises questions:

- Why these three factors?
- Why are electric charges quantized?
- Why do quarks and leptons fit together generation by generation?
- Could the three gauge couplings meet at high energy?

$\boxed{\text{Grand unification tries to embed the SM group in one simple group.}}$

Running couplings

Quantum corrections make gauge couplings depend on energy scale:

$$\alpha_i = \alpha_i(\mu).$$

Qualitative behaviour:

- QCD coupling decreases at high energy: asymptotic freedom.
- Electroweak couplings run differently.
- Extrapolated couplings come close to one another at a very high scale.

$$\boxed{\text{Near-unification suggests a deeper gauge structure.}}$$

Couplings almost in the SM unify (exactly in MSSM!) at

$$\boxed{10^{15}\ \mathrm{GeV}.}$$

The simplest idea: SU(5)

The smallest simple group containing the SM gauge group is

$$\boxed{SU(5).}$$

One can embed

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5).$$

In the defining representation, the first three components may carry colour and the last two weak isospin.

A hypercharge generator can be chosen proportional to

$$\boxed{Y \sim \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right).}$$

Important consequence

Hypercharge normalization is no longer arbitrary: charge quantization becomes natural.

Quarks and leptons in common multiplets

In grand unification, quarks and leptons can sit in common multiplets.

For one generation in minimal $SU(5)$, the SM fermions fit schematically into

$$\boxed{\bar{\mathbf{5}} \oplus \mathbf{10}.}$$

This is suggestive because one generation contains exactly the right quantum numbers.

Conceptual gain

The pattern of SM charges looks less accidental.

Cost

New heavy gauge bosons appear, and they can mediate baryon-number-violating processes.

Proton decay as a smoking gun

Grand unified theories generically relate quarks and leptons.

Therefore they can generate processes such as

$$\boxed{p \to e^+\pi^0, \qquad p \to \bar{\nu}K^+.}$$

These processes are not allowed in the renormalizable SM. They do arise from dimension ≥ 6 operators in SM EFT.

Phenomenological lesson

The non-observation of proton decay pushes the unification scale very high and strongly constrains simple GUT models.

GUTs are attractive, but highly constrained.

Flavour and CP as precision tests

Flavour physics is a powerful test of the SM.

The SM contains

$$\boxed{V_{\mathrm{CKM}}}$$

with one irreducible CP-violating phase for three generations.

But the observed baryon asymmetry requires more:

$$\boxed{\text{SM CKM CP violation is not enough for baryogenesis.}}$$

Thus flavour and CP violation are both

- a triumph of the SM, and
- a sensitive place to search for new physics.

A compact list of open questions

At the end of the course, the open questions can be organized as follows:

Observation or puzzle	Possible interpretation
Neutrino masses	New lepton-number dynamics
Dark matter	New stable neutral particle
Baryon asymmetry	New CP violation or new dynamics
Hierarchy problem	New physics near or above the TeV scale
Charge quantization	Grand unification?
Gravity	QFT must be extended

The SM is not the end of the story; it is the map of where to look next.

My five-lecture story

$\boxed{\text{QM} + \text{relativity} \Rightarrow \text{quantum fields}}$

$\boxed{\text{local gauge symmetry} \Rightarrow \text{interactions}}$

$\boxed{SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow \text{strong and electroweak forces}}$

$\boxed{\text{Higgs mechanism} \Rightarrow W^{\pm}, Z \text{ masses and the Higgs boson}}$

$\boxed{\text{Yukawa couplings} \Rightarrow \text{fermion masses and flavour}}$

$\boxed{\text{EFT} \Rightarrow \text{a systematic language for BSM physics}}$

Final message

The Standard Model

A renormalizable gauge quantum field theory of the known non-gravitational particles.

Its success

It describes a huge range of data, including precision electroweak and flavour measurements.

Its limitation

It does not explain neutrino masses, dark matter, baryogenesis, the electroweak hierarchy, or gravity.

The SM is both a triumph and a starting point.

Reading for this lecture

General:

- D. Griffiths, Introduction to Elementary Particle Physics, 2008.
- M. Peskin, Concepts of Elementary Particle Physics, Oxford Master Series, 2019.
- B. Gripaios, From QM to the SM, arXiv:2005.06355
- A. Pich, The SM of Electroweak interactions, arXiv:1201.0537

SM+EFT Feynman Rules

- A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek and K. Suxho, "Feynman rules for the Standard Model Effective Field Theory in R_ξ -gauges," JHEP **06** (2017), 143 arXiv:1704.03888.

Supersymmetry:

- J. Wess and J. Bagger, Supersymmetry and Supergravity, Princeton Univ. Press, 1990.
- H. Dreiner, H. Haber, S. Martin, From Spinors to Supersymmetry, Cambridge Univ. Press., 2023.

Tutorial Problem 1: Weinberg operator scale

The dimension-five neutrino mass operator gives the estimate

$$m_\nu \sim \frac{v^2}{\Lambda}.$$

Take

$$v = 246\ \mathrm{GeV}, \qquad m_\nu = 0.05\ \mathrm{eV}.$$

Questions

1. Convert m_ν to GeV.
2. Estimate Λ.
3. Compare your answer with the electroweak scale and the Planck scale.

Lesson

A tiny neutrino mass can indicate an enormous new-physics scale.

Tutorial Problem 2: Oscillation baseline

For two-flavour neutrino oscillations,

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\,\sin^2\left(1.27\frac{\Delta m^2[\mathrm{eV}^2]L[\mathrm{km}]}{E[\mathrm{GeV}]}\right).$$

Take

$$\Delta m^2 = 2.4 \times 10^{-3}\ \mathrm{eV}^2, \qquad E = 1\ \mathrm{GeV}.$$

Questions

1. Find the baseline L for the first oscillation maximum.
2. How does the answer change if E is doubled?

Hint

The first maximum occurs when the argument of the second sine is $\pi/2$.

Tutorial Problem 3: Naturalness estimate

Suppose a heavy scale Λ contributes to the Higgs mass as

$$\delta m_h^2 \sim \frac{\Lambda^2}{16\pi^2}.$$

Questions

1. Estimate δm_h for $\Lambda = 1\ \mathrm{TeV}$.
2. Estimate δm_h for $\Lambda = 10^{15}\ \mathrm{GeV}$.
3. Compare both with $m_h \simeq 125\ \mathrm{GeV}$.

Lesson

Very high scales make the lightness of the Higgs look unnatural unless there is a protection mechanism or a cancellation.

Solutions: Problem 1

Convert

$$0.05 \text{ eV} = 5 \times 10^{-11} \text{ GeV}.$$

Then

$$\Lambda \sim \frac{v^2}{m_\nu} = \frac{(246 \text{ GeV})^2}{5 \times 10^{-11} \text{ GeV}} \sim 1.2 \times 10^{15} \text{ GeV}.$$

$$\boxed{\Lambda \sim 10^{15} \text{ GeV}.}$$

This is far above the electroweak scale and not very far below typical grand-unification scales.

Solutions: Problem 2

The first maximum satisfies

$$1.27\frac{\Delta m^2 L}{E} = \frac{\pi}{2}.$$

Thus

$$L = \frac{\pi E}{2 \times 1.27\Delta m^2}.$$

For $E = 1\ \mathrm{GeV}$ and $\Delta m^2 = 2.4 \times 10^{-3}\ \mathrm{eV}^2$,

$$L \simeq \frac{3.14}{2 \times 1.27 \times 2.4 \times 10^{-3}}\ \mathrm{km} \simeq 5.2 \times 10^2\ \mathrm{km}.$$

$$\boxed{L \sim 500\ \mathrm{km.}}$$

If E is doubled, L is doubled.

Solutions: Problem 3

Using

$$\delta m_h^2 \sim \frac{\Lambda^2}{16\pi^2},$$

we estimate

$$\delta m_h \sim \frac{\Lambda}{4\pi}.$$

For $\Lambda = 1\ \mathrm{TeV}$,

$$\delta m_h \sim \frac{1000\ \mathrm{GeV}}{12.6} \sim 80\ \mathrm{GeV}.$$

This is comparable to $m_h \simeq 125\ \mathrm{GeV}$.
For $\Lambda = 10^{15}\ \mathrm{GeV}$,

$$\delta m_h \sim 10^{14}\ \mathrm{GeV},$$

which is enormously larger than the observed Higgs mass.

$\boxed{\text{This is the hierarchy problem in one line.}}$

End of the course

$\boxed{\text{The Standard Model is the beginning of modern particle physics.}}$

Thank you!