Lecture 4: Electroweak Symmetry Breaking, the Higgs Boson, and the EFT View

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Introductory Lectures on the Standard Model

Purpose of Lecture 4

Central question

How can the weak gauge bosons be massive while the photon remains massless?

Lecture 3 left us with the electroweak gauge symmetry

$$SU(2)_L \times U(1)_Y,$$

and with the problem that gauge invariance forbids explicit gauge-boson masses.

Today we follow the chain

$$\boxed{\text{Spontaneous SB} \;\Rightarrow\; \text{Higgs mechanism} \;\Rightarrow\; W^{\pm}, Z \text{ masses} \;\Rightarrow\; \text{Higgs boson}}$$

and then add a modern lesson: the SM as an Effective Field Theory (SMEFT).

Roadmap of the lecture

1. Why masses are a problem for gauge theories
2. Spontaneous symmetry breaking in a scalar theory
3. Goldstone bosons and the abelian Higgs mechanism
4. Electroweak symmetry breaking: $SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{em}}$
5. Gauge-boson masses and the Weinberg angle
6. Fermion masses and Higgs couplings
7. The Higgs boson phenomenology at a qualitative level
8. Renormalization and effective field theory

Boundary

The goal is not a full QFT course: the emphasis is on the SM logic and the high-value formulae.

Gauge symmetry versus mass terms

In QED, gauge invariance forbids a photon mass term:

$$\Delta\mathcal{L} = \frac{1}{2} m_\gamma^2 A_\mu A^\mu.$$

Similarly, before electroweak symmetry breaking we cannot write

$$\frac{1}{2} M^2 W_\mu^i W^{i\mu}, \qquad \frac{1}{2} M^2 B_\mu B^\mu.$$

But Nature gives us

$$M_W \simeq 80 \ \mathrm{GeV}, \qquad M_Z \simeq 91 \ \mathrm{GeV}, \qquad M_\gamma = 0.$$

We need gauge-invariant dynamics whose vacuum hides part of the symmetry.

The idea in one sentence

Spontaneous symmetry breaking

The Lagrangian is symmetric, but the vacuum chosen by the system is not.

For the electroweak theory the desired pattern is

$$\boxed{SU(2)_L \times U(1)_Y \longrightarrow U(1)_{\mathrm{em}}.}$$

Consequences:

- the broken generators give mass to $W^{\pm}$ and Z,
- the unbroken generator gives the massless photon,
- a physical scalar remains: the Higgs boson.

Warm-up: a complex scalar field

Consider a complex scalar with

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi),$$

where

$$\boxed{V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4, \qquad \lambda > 0.}$$

This theory has a global symmetry

$$\phi \to e^{i\alpha}\phi.$$

If $m^2 > 0$, the minimum is

$$\phi = 0.$$

If $m^2 < 0$, the minimum is no longer at the origin.

The Mexican-hat minimum

For $m^2 < 0$, minimize

$$V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4.$$

The minimum satisfies

$$|\phi|^2 = -\frac{m^2}{2\lambda} \equiv \frac{v^2}{2}.$$

Thus

$$\boxed{|\phi| = \frac{v}{\sqrt{2}}, \qquad v^2 = -\frac{m^2}{\lambda}.}$$

There is a whole circle of degenerate vacua.

Choose one, for example

$$\boxed{\langle\phi\rangle = \frac{v}{\sqrt{2}}.}$$

The symmetry is present in the Lagrangian but not manifest in the chosen vacuum.

Fluctuations: radial and angular modes

Choose the vacuum along the real direction and write

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}} \left[v + \phi_1(x) + i\phi_2(x) \right].}$$

Substitution into the potential gives, at quadratic order,

$$\phi_1 : \quad m^2_{\phi_1} = -2m^2 = 2\lambda v^2,$$

while

$$\phi_2 : \quad m^2_{\phi_2} = 0.$$

Goldstone theorem

For every broken continuous global generator, there is a massless scalar boson.

Gauge the symmetry: the abelian Higgs model

Promote the global phase symmetry to a local one:

$$\phi(x) \to e^{i\alpha(x)}\phi(x).$$

Introduce

$$\boxed{D_\mu = \partial_\mu + ieA_\mu.}$$

The Lagrangian becomes

$$\boxed{\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - m^2|\phi|^2 - \lambda|\phi|^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.}$$

When $m^2 < 0$, the scalar develops a VEV:

$$\langle\phi\rangle = \frac{v}{\sqrt{2}}.$$

Gauge-boson mass from the scalar kinetic term

The key term is

$$(D_\mu \phi)^*(D^\mu \phi).$$

In unitary gauge we write

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}}\left[v + h(x)\right].}$$

Then

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left[\partial_\mu h + ieA_\mu (v + h)\right].$$

Hence the Lagrangian contains

$$\boxed{(D_\mu \phi)^*(D^\mu \phi) \supset \frac{1}{2} e^2 v^2 A_\mu A^\mu.}$$

Therefore

$$\boxed{m_A = e\, v.}$$

Where did the Goldstone boson go?

A useful parametrization is

$$\phi(x) = \frac{1}{\sqrt{2}}\,[v + h(x)]\, e^{i\chi(x)/v}.$$

For a local symmetry we can gauge away $\chi(x)$:

$$\boxed{\text{unitary gauge: } \chi(x) = 0.}$$

Degree-of-freedom counting:

$$\textbf{before:}\ \text{massless vector } (2) + \text{Goldstone } (1) = 3,$$

$$\textbf{after:}\ \text{massive vector } (3) = 3$$

The Goldstone boson is eaten and becomes the longitudinal vector polarization.

Non-abelian generalization

Let a scalar field in representation r of a group G acquire a VEV v.

The gauge-boson mass matrix comes from

$$(D_\mu \Phi)^\dagger (D^\mu \Phi).$$

Schematically,

$$\boxed{(m^2)^{ab} \sim g^2\, v^\dagger\, T_r^a\, T_r^b v.}$$

If

$$T^a v = 0,$$

then the corresponding gauge boson remains massless.

If

$$T^a v \neq 0,$$

then the corresponding generator is broken and its gauge boson becomes massive.

$$\boxed{G \to H : \quad \text{unbroken generators span } H.}$$

The electroweak theory before breaking

Gauge group:

$$\boxed{SU(2)_L \times U(1)_Y.}$$

Gauge bosons:

$$W_\mu^1,\ W_\mu^2,\ W_\mu^3, \qquad B_\mu.$$

Higgs field:

$$\boxed{H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \qquad Y_H = \frac{1}{2}.}$$

Covariant derivative:

$$\boxed{D_\mu H = \left(\partial_\mu + ig\frac{\sigma^i}{2} W_\mu^i + ig'\frac{1}{2} B_\mu\right) H.}$$

The Higgs potential

The SM Higgs potential is

$$\boxed{V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2, \qquad \mu^2 > 0, \quad \lambda > 0.}$$

The minimum satisfies

$$\sqrt{H^\dagger H} = \frac{v}{\sqrt{2}}, \qquad v^2 = \frac{\mu^2}{\lambda}.$$

Choose the vacuum

$$\boxed{\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}.}$$

This is a choice of orientation in the Higgs field space.

$\boxed{\text{It preserves electric charge.}}$

The unbroken generator

The electric charge generator is

$$\boxed{Q = T_3 + Y.}$$

For the Higgs VEV,

$$T_3 \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix},$$

while

$$Y \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = +\frac{1}{2} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}.$$

Thus

$$\boxed{Q \langle H \rangle = 0.}$$

Therefore

$$\boxed{SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{em}},}$$

and the photon remains massless.

Mass terms from $(D_\mu H)^\dagger(D^\mu H)$

Set

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}.$$

Then the gauge-boson mass terms are

$$\boxed{(D_\mu \langle H \rangle)^\dagger (D^\mu \langle H \rangle) = \frac{v^2}{8} \left[g^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + (g W_\mu^3 - g' B_\mu)^2 \right].}$$

This equation contains all the main electroweak mass predictions.

Charged gauge bosons

Define

$$\boxed{W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right).}$$

The charged part of the mass term becomes

$$\frac{g^2 v^2}{4} W_\mu^+ W^{-\mu}.$$

Therefore

$$\boxed{M_W = \frac{gv}{2}.}$$

The charged weak bosons are the gauge bosons associated with the broken charged generators.

Neutral gauge bosons: Z and γ

The neutral mass term is

$$\frac{v^2}{8}(gW_\mu^3 - g' B_\mu)^2.$$

Define the Weinberg angle by

$$\boxed{\tan\theta_W = \frac{g'}{g}.}$$

The physical fields are

$$\boxed{Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu,}$$

$$\boxed{A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu.}$$

Only Z_μ appears in the mass term; A_μ is massless.

Tree-level electroweak mass relations

The neutral masses are

$$\boxed{M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v, \qquad M_\gamma = 0.}$$

Using

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}},$$

we obtain

$$\boxed{M_W = M_Z \cos\theta_W.}$$

The electromagnetic coupling is

$$\boxed{e = g \sin\theta_W = g' \cos\theta_W.}$$

High-value result

The Higgs representation $(1, 2, +1/2)$ gives exactly the observed pattern $W^\pm, Z$ massive and γ massless.

Counting degrees of freedom

Before symmetry breaking:

$$H : 4 \text{ real scalars}, \qquad W^1, W^2, W^3, B : 4 \times 2 = 8,$$

so

$$4 + 8 = 12.$$

After symmetry breaking:

$$W^+, W^-, Z : 3 \times 3 = 9, \qquad \gamma : 2, \qquad h : 1,$$

so

$$9 + 2 + 1 = 12.$$

$\boxed{\text{The three Goldstones become the longitudinal modes of } W^\pm, Z.}$

Fermion masses: the remaining problem

Because the weak interaction is chiral,

$$\psi_L \quad \text{and} \quad \psi_R$$

transform differently under $SU(2)_L \times U(1)_Y$.

Therefore a direct mass term

$$-m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

is not gauge invariant before symmetry breaking.

Solution

Fermion masses come from gauge-invariant Yukawa interactions with the Higgs field.

Yukawa interactions

For one generation,

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}.$$

Define the conjugate Higgs doublet

$$\boxed{H^c = i\sigma^2 H^*.}$$

Yukawa terms:

$$\boxed{\mathcal{L}_Y \supset -\lambda^u \bar{q}_L H^c u_R - \lambda^d \bar{q}_L H d_R - \lambda^e \bar{\ell}_L H e_R + \mathrm{h.c.}.}$$

In unitary gauge,

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}.$$

Masses and Higgs–fermion couplings

Substituting the Higgs VEV gives

$$\boxed{m_u = \frac{\lambda^u v}{\sqrt{2}}, \qquad m_d = \frac{\lambda^d v}{\sqrt{2}}, \qquad m_e = \frac{\lambda^e v}{\sqrt{2}}.}$$

Keeping the Higgs fluctuation h, one obtains

$$\boxed{\mathcal{L} \supset -\frac{m_f}{v} h \bar{f} f.}$$

Physical message

The same interaction gives the fermion mass and the Higgs coupling:

$$\boxed{g_{hff} = \frac{m_f}{v}.}$$

It just works!

Three generations: a brief preview

For three generations, Yukawa couplings become matrices:

$$\mathcal{L}_Y = -\bar{q}_L Y_u H^c u_R - \bar{q}_L Y_d H d_R - \bar{\ell}_L Y_e H e_R + \mathrm{h.c.}.$$

After symmetry breaking,

$$M_u = \frac{v}{\sqrt{2}} Y_u, \qquad M_d = \frac{v}{\sqrt{2}} Y_d, \qquad M_e = \frac{v}{\sqrt{2}} Y_e.$$

The matrices are not generally diagonal.

$$\boxed{\text{Their diagonalization leads to flavour physics and CKM mixing.}}$$

(Unfortunately, detailed flavour physics is kept out of my lectures.)

The SM field content after the construction

The minimal SM contains fields with representations under

$$SU(3)_c \times SU(2)_L \times U(1)_Y.$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
G	8	1	0
W	1	3	0
B	1	1	0
$q_L = (u_L, d_L)^T$	3	2	$1/6$
u_R	3	1	$2/3$
d_R	3	1	$-1/3$
$\ell_L = (\nu_L, e_L)^T$	1	2	$-1/2$
e_R	1	1	-1
H	1	2	$1/2$

The physical Higgs boson

In unitary gauge,

$$\boxed{H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}.}$$

The three Goldstones have been eaten by $W^{\pm}$ and Z.

One real scalar remains:

$$\boxed{h(x) = \text{the Higgs boson.}}$$

From the potential,

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2,$$

we get

$$\boxed{m_h^2 = 2\lambda v^2.}$$

For $m_h \simeq 125\ \mathrm{GeV}$,

$$\lambda \simeq 0.13.$$

Higgs couplings to gauge bosons

Replace

$$v \to v + h$$

in the gauge-boson mass terms.

For the charged bosons:

$$\mathcal{L} \supset M_W^2 \left(1 + \frac{h}{v}\right)^2 W_\mu^+ W^{-\mu}.$$

For the neutral boson:

$$\mathcal{L} \supset \frac{1}{2} M_Z^2 \left(1 + \frac{h}{v}\right)^2 Z_\mu Z^\mu.$$

Thus

$$\boxed{g_{hWW} = \frac{2M_W^2}{v}, \qquad g_{hZZ} = \frac{2M_Z^2}{v}.}$$

The Higgs coupling remembers the origin of mass.

Higgs self-interactions

Expanding the Higgs potential in unitary gauge gives

$$\mathcal{L} \supset -\frac{m_h^2}{2}h^2 - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4.$$

So the Higgs sector predicts

- a physical scalar mass m_h,
- cubic self-interaction,
- quartic self-interaction.

Experimental frontier

Measuring the Higgs self-coupling tests the shape of the Higgs potential itself.

Qualitative Higgs phenomenology

The Higgs couplings scale with mass:

$$\boxed{g_{hff} = \frac{m_f}{v}, \qquad g_{hVV} \propto \frac{M_V^2}{v}.}$$

Therefore the Higgs prefers the heaviest available final states.

For a light Higgs near 125 GeV:

$$h \to b\bar{b}, \quad h \to WW^*, \quad h \to ZZ^*, \quad h \to \tau^+\tau^-, \quad h \to \gamma\gamma.$$

Loop-induced Higgs couplings

The Higgs carries neither colour nor electric charge.

So why do we have

$$h \to gg\,, \qquad h \to \gamma\gamma \qquad ?$$

They arise from loops of heavy SM particles:

- $h \to gg$: mainly top-quark loops,
- $h \to \gamma\gamma$: mainly W-boson and top-quark loops.

$\boxed{\text{Loop effects are small but crucial for LHC Higgs production and discovery.}}$

Why renormalization appears

Loop diagrams in QFT often contain integrals over arbitrary momenta.

Example: an electron self-energy correction behaves schematically as

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\not{k}}{k^4},$$

which is ultraviolet divergent.

Old view

Absorb divergences into the parameters of the Lagrangian: masses, charges, fields.

Modern view

A QFT is an effective description valid up to some scale. The UV sensitivity tells us how low-energy physics depends on high-energy physics.

Dimensional analysis of fields

In units $\hbar = c = 1$, the action is dimensionless:

$$S = \int \mathrm{d}^4x\,\mathcal{L}, \qquad [\mathcal{L}] = 4.$$

From kinetic terms:

$$\boxed{[\text{scalar}] = 1, \qquad [\text{gauge field}] = 1, \qquad [\text{fermion}] = \frac{3}{2}.}$$

Gauge and Yukawa couplings are dimensionless:

$$[g] = [g'] = [g_s] = [y_f] = 0.$$

Masses have dimension one:

$$[m] = 1.$$

$$\boxed{\text{The SM contains only operators of mass dimension } \leq 4.}$$

Renormalizable versus non-renormalizable

A coupling multiplying an operator $\mathcal{O}_d$ has dimension

$$[C_d] = 4 - d.$$

If $d \leq 4$, the coupling has non-negative mass dimension.

If $d > 4$, the coupling has negative mass dimension and is naturally written as

$$\frac{C_d}{\Lambda^{d-4}}\mathcal{O}_d.$$

non-renormalizable terms are not wrong; they signal an effective theory.

Fermi theory as the prototype EFT

Before the electroweak theory, beta decay was described by a four-fermion interaction:

$$\mathcal{L}_{\text{Fermi}} \sim -G_F(\bar{\psi}\psi)(\bar{\psi}\psi).$$

Since fermions have dimension $3/2$, this operator has dimension six:

$$[(\bar{\psi}\psi)(\bar{\psi}\psi)] = 6, \qquad [G_F] = -2.$$

The scale is roughly

$$G_F \sim \frac{1}{v^2} \sim \frac{1}{(246\ \text{GeV})^2}.$$

The non-renormalizable Fermi theory pointed to W,Z-bosons at around $100\ \text{GeV}$.

The Standard Model as an effective theory

The modern viewpoint is

$$\boxed{\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i^{(5)}}{\Lambda} Q_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \cdots .}$$

Here Λ is the scale of new physics.

At energies $E \ll \Lambda$, higher-dimensional effects are suppressed:

$$\left(\frac{E}{\Lambda}\right)^{d-4} .$$

Practical meaning

Precision measurements can probe virtual effects of new heavy particles, even if those particles are not directly produced.

Gravity and the EFT viewpoint

Gravity can also be treated as an effective field theory.

The Einstein–Hilbert action contains

$$S \sim M_P^2 \int \mathrm{d}^4x \sqrt{-g}\, R.$$

Perturbatively, gravitational interactions involve couplings suppressed by powers of

$$M_P \sim 10^{19}\ \mathrm{GeV}.$$

Key lesson

Non-renormalizable does not mean useless. It means: do not ask the EFT questions beyond its cutoff.

This prepares the ground for neutrino masses and BSM physics in Lecture 5.

What Lecture 4 established

$\boxed{\text{The Higgs VEV hides } SU(2)_L \times U(1)_Y \text{ and preserves } U(1)_{\mathrm{em}}.}$

$$\boxed{M_W = \frac{gv}{2}, \qquad M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v, \qquad M_\gamma = 0.}$$

$$\boxed{M_W = M_Z \cos\theta_W, \qquad e = g \sin\theta_W = g' \cos\theta_W.}$$

$$\boxed{m_f = \frac{y_f\, v}{\sqrt{2}}, \qquad g_{hff} = \frac{m_f}{v}.}$$

$\boxed{\mathcal{L}_{\mathrm{SM}} \text{ works as a renormalizable QFT, but can be viewed as an EFT.}}$

Bridge to Lecture 5

We have now built the SM gauge and Higgs structure.

Lecture 5 will focus on:

1. neutrino masses,
2. hierarchy problem and grand unification,
3. the open questions beyond the Standard Model.

Final message of Lecture 4

The Higgs mechanism is not merely a mass-generating trick. It is the mechanism that makes the electroweak gauge theory look like the world we observe.

Reading for this lecture

- D. Griffiths, Introduction to Elementary Particle Physics, 2008.
- M. Peskin, Concepts of Elementary Particle Physics, Oxford Master Series, 2019.
- B. Gripaios, From QM to the SM, arXiv:2005.06355

More advanced textbooks:

- M. Peskin, Intro to QFT, Addison-Wesley, 1995.
- D. Schwartz, Quantum Field Theory, Cambridge University Press, 2016?

Of course, the Bible of QFT (and everything!) are the books:

- S. Weinberg, Quantum Theory of Fields, Cambridge University Press. Vol.1 and Vol 2.

but do not start directly from those!

Problem 1: The Mexican-hat potential

Consider a complex scalar field ϕ with potential

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \qquad \lambda > 0.$$

Questions

1. Show that for $\mu^2 > 0$, the minimum is at
$$\phi = 0.$$
2. Show that for $\mu^2 < 0$, the minimum satisfies
$$\phi^\dagger \phi = -\frac{\mu^2}{2\lambda}.$$
3. Define
$$\phi(x) = \frac{1}{\sqrt{2}} \left[v + h(x) + i\chi(x) \right].$$
Find v in terms of μ^2 and λ, and identify which field is massive and which is massless.

Problem 1: Expected lesson

For $\mu^2 < 0$, the vacuum satisfies

$$|\phi_0| = \frac{v}{\sqrt{2}}, \qquad v^2 = -\frac{\mu^2}{\lambda}.$$

Expanding around one vacuum direction,

$$\phi(x) = \frac{1}{\sqrt{2}}\left[v + h(x) + i\chi(x)\right],$$

one finds

$$h: \text{ massive radial mode,} \qquad \chi: \text{ massless Goldstone mode.}$$

Main message

$\boxed{\text{A symmetric Lagrangian may have a non-symmetric vacuum.}}$

$\boxed{\text{A broken global continuous symmetry gives a Goldstone boson.}}$

Problem 2: Gauge boson mass in the Abelian Higgs model

Consider a complex scalar field coupled to a $U(1)$ gauge field:

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi),$$

where

$$D_\mu = \partial_\mu + ieA_\mu.$$

Assume that the scalar field develops a vacuum expectation value. In unitary gauge,

$$\phi(x) = \frac{1}{\sqrt{2}} \left[v + h(x) \right].$$

Questions

1. Compute $D_\mu \phi$.
2. Show that the scalar kinetic term contains a gauge-boson mass term of the form
$$\frac{1}{2} M_A^2 A_\mu A^\mu.$$
3. Find M_A.

Problem 2: Expected result

In unitary gauge,

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h).$$

Therefore

$$D_\mu \phi = \frac{1}{\sqrt{2}} \left[\partial_\mu h + ieA_\mu (v + h) \right].$$

The kinetic term contains

$$(D_\mu \phi)^\dagger (D^\mu \phi) \supset \frac{1}{2} e^2 v^2 A_\mu A^\mu.$$

Comparing with

$$\frac{1}{2} M_A^2 A_\mu A^\mu,$$

we obtain

$$\boxed{M_A = ev.}$$

Main message: In a gauge theory, the Goldstone boson is eaten by the gauge field.

Problem 3: Electroweak gauge-boson masses

In the Standard Model, the Higgs doublet in unitary gauge is

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \qquad Y_\Phi = \frac{1}{2}.$$

The electroweak covariant derivative is

$$D_\mu = \partial_\mu + ig\frac{\tau^i}{2} W_\mu^i + ig' Y B_\mu.$$

Questions

1. Set $h = 0$ and compute

$$(D_\mu \langle \Phi \rangle)^\dagger (D^\mu \langle \Phi \rangle).$$

2. Show that it gives the electroweak gauge-boson mass terms.

Problem 3: Intermediate result

Using

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad Y_\Phi = \frac{1}{2},$$

one finds

$$(D_\mu \langle \Phi \rangle)^\dagger (D^\mu \langle \Phi \rangle) = \frac{v^2}{8} \left[g^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + (g W_\mu^3 - g' B_\mu)^2 \right].$$

Define

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right).$$

Then the charged gauge boson mass is

$$\boxed{M_W = \frac{gv}{2}.}$$

Problem 3: Neutral gauge bosons

The neutral mass term is

$$\frac{v^2}{8}(gW_\mu^3 - g' B_\mu)^2.$$

Define

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu,$$
$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu,$$

with

$$\tan\theta_W = \frac{g'}{g}.$$

Then

$$\boxed{M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v,} \qquad \boxed{M_\gamma = 0.}$$

Therefore,

$$\boxed{M_W = M_Z \cos\theta_W.}$$

Problem 3: Expected lesson

The Higgs vacuum breaks

$$SU(2)_L \times U(1)_Y$$

but preserves

$$U(1)_{\mathrm{em}}.$$

The unbroken generator is

$$Q = T_3 + Y.$$

Thus

$$\boxed{W^{\pm} \text{ and } Z \text{ become massive,}}$$

while

$$\boxed{\gamma \text{ remains massless.}}$$

Main message

$$\boxed{SU(2)_L \times U(1)_Y \longrightarrow U(1)_{\mathrm{em}}.}$$

$$\boxed{\text{The photon is massless because electromagnetism is unbroken.}}$$