Lecture 3: Gauge Field Theories and the Weak Interaction

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Introductory Lectures on the Standard Model

Purpose of Lecture 3

Central question

Why do the interactions of elementary particles have the form they do?

Lecture 2 gave us fields, particles, propagators and vertices.

Today we ask why the vertices are not arbitrary:

$$\boxed{\text{local gauge symmetry fixes the interactions.}}$$

We then build the route

$$\boxed{\text{QED} \;\longrightarrow\; \text{non-abelian gauge theory} \;\longrightarrow\; \text{QCD and } SU(2)_L \times U(1)_Y.}$$

Roadmap of the lecture

1. QED as a gauge theory
2. Feynman rules from a gauge-invariant Lagrangian
3. Minimal group theory for non-abelian gauge theories
4. Yang–Mills theory and gauge-boson self-interactions
5. QCD as an $SU(3)_c$ gauge theory
6. Weak interactions and $SU(2)$
7. Parity violation, chirality and $SU(2)_L \times U(1)_Y$

Important boundary of this lecture

We stop before spontaneous symmetry breaking. Masses and the Higgs mechanism are Lecture 4.

From Lecture 2: QED interaction

For electrons and photons we used

$$\mathcal{L}_{\text{int}}^{\text{QED}} = -eA_\mu \bar{\psi}\gamma^\mu\psi.$$

This gives the vertex

$$\boxed{\gamma_\mu \bar{e} e : \qquad -ie\gamma^\mu.}$$

Question

Why exactly this interaction? Why not any Lorentz-invariant term we can write?

The answer is gauge invariance.

Local $U(1)$ symmetry

Start with the local phase transformation

$$\psi(x) \to e^{ie\alpha(x)}\psi(x).$$

Ordinary derivatives do not transform covariantly:

$$\partial_\mu \psi \to e^{ie\alpha(x)}\left(\partial_\mu + ie\partial_\mu \alpha\right)\psi.$$

Introduce a gauge field and define

$$\boxed{D_\mu = \partial_\mu + ieA_\mu.}$$

If

$$\boxed{A_\mu \to A_\mu - \partial_\mu \alpha,}$$

then

$$\boxed{D_\mu \psi \to e^{ie\alpha(x)} D_\mu \psi.}$$

QED Lagrangian

The gauge-invariant QED Lagrangian is

$$\boxed{\mathcal{L}_{\mathrm{QED}} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Expanding the covariant derivative,

$$\bar{\psi} i\gamma^\mu D_\mu \psi = \bar{\psi} i\gamma^\mu \partial_\mu \psi - eA_\mu \bar{\psi}\gamma^\mu\psi.$$

Thus

$$\boxed{\text{the QED vertex follows from local } U(1) \text{ gauge invariance.}}$$

Why the photon is massless

A photon mass term would be

$$\Delta \mathcal{L} = \frac{1}{2} m_\gamma^2 A_\mu A^\mu.$$

But under a gauge transformation,

$$A_\mu A^\mu \not\to A_\mu A^\mu.$$

So local $U(1)$ gauge invariance forbids

$$\boxed{m_\gamma \neq 0.}$$

Lesson

Gauge symmetry is not merely aesthetic. It removes possible terms from the Lagrangian.

QED Feynman rules: basic ingredients

In momentum space, the main tree-level QED rules are:

Photon propagator:

$$\boxed{\frac{-ig_{\mu\nu}}{q^2 + i\epsilon}}$$

Electron propagator:

$$\boxed{\frac{i}{\not{q} - m + i\epsilon}}$$

Electron-photon vertex:

$$\boxed{-ie\gamma^{\mu}}$$

External fermions are represented by spinors $u, \bar{u}, v, \bar{v}$; external photons by polarization vectors ϵ_{μ}.

Example: electron–electron scattering

For

$$e^- e^- \to e^- e^-,$$

there are two tree-level diagrams because the final electrons are identical.

Schematically,

$$\mathcal{M} = \mathcal{M}_t - \mathcal{M}_u.$$

The relative minus sign is a fermionic effect.

Takeaway

The Feynman rules are not arbitrary recipes. They encode the fields, symmetries and statistics of the theory.

Why group theory enters

QED is based on

$$\psi \to e^{ie\alpha(x)}\psi.$$

This is a $U(1)$ gauge theory.

Can we build theories where the transformation is a matrix?

$$\psi(x) \to U(x)\psi(x), \qquad U(x) = \exp\left[ig\alpha^a(x)T^a\right].$$

Answer

Yes: Yang–Mills gauge theories.

Lie algebra near the identity

For a continuous group, study transformations close to the identity:

$$U = 1 + ig\alpha^a T^a + \cdots .$$

The generators satisfy

$$\boxed{[T^a, T^b] = if^{abc} T^c.}$$

- T^a: generators of the group
- f^{abc}: structure constants
- Abelian case: $f^{abc} = 0$
- Non-abelian case: $f^{abc} \neq 0$

Non-commuting generators are the origin of gauge-boson self-interactions.

Representations

Matter fields transform in representations of the gauge group.

For example:

$$\psi_i \to U_{ij}\psi_j.$$

The matrices T^a depend on the representation.

Examples:

- Electron in QED: one-dimensional representation of $U(1)$
- Quarks in QCD: triplets of $SU(3)_c$
- Left-handed leptons: doublets of $SU(2)_L$

Guiding principle

To build a gauge theory, specify the gauge group and the representations of all matter fields.

Non-abelian covariant derivative

For matter transforming as

$$\psi \to U(x)\psi,$$

we define

$$\boxed{D_\mu = \partial_\mu + igA_\mu^a T^a.}$$

The gauge field can be written as a matrix:

$$A_\mu = A_\mu^a T^a.$$

It transforms so that

$$\boxed{D_\mu \psi \to U(x) D_\mu \psi.}$$

For infinitesimal transformations, the gauge fields themselves transform in the adjoint representation.

Non-abelian field strength

The field strength is obtained from the commutator of covariant derivatives:

$$[D_\mu, D_\nu] = igF^a_{\mu\nu} T^a.$$

This gives

$$\boxed{F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc} A^b_\mu A^c_\nu.}$$

Compare with QED:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The extra term is the non-abelian novelty.

Yang–Mills Lagrangian

The gauge-field kinetic term is

$$\boxed{\mathcal{L}_{\text{YM}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}.}$$

Because

$$F^a_{\mu\nu} = \partial A + gAA,$$

we have

$$F^2 \sim (\partial A)^2 + g(\partial A)AA + g^2 A^4.$$

Therefore non-abelian gauge theories contain

$$\boxed{AAA \quad \text{and} \quad AAAA}$$

vertices.

Physical meaning

Non-abelian gauge bosons carry the gauge charge and interact with each other.

QCD: the strong force as $SU(3)_c$

QCD is a gauge theory with group

$$\boxed{SU(3)_c.}$$

Quarks carry colour:

$$q = \begin{pmatrix} q_r \\ q_b \\ q_g \end{pmatrix}.$$

There are

$$\dim SU(3) = 3^2 - 1 = 8$$

gauge bosons:

$$\boxed{G^a_\mu, \qquad a = 1, \dots, 8.}$$

These are the gluons.

QCD Lagrangian

The QCD covariant derivative is

$$\boxed{D_\mu = \partial_\mu + i g_s G_\mu^a T^a, \qquad T^a = \frac{\lambda^a}{2}.}$$

The field strength is

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c.$$

The Lagrangian is

$$\boxed{\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \sum_f \bar{q}_f (i\not{D} - m_f) q_f.}$$

What is special about QCD?

QCD has the vertices

$$\boxed{q\bar{q}g\,, \qquad ggg\,, \qquad gggg\,.}$$

The gluon self-interactions lead to qualitatively new physics:

- asymptotic freedom at short distances;
- confinement at long distances;
- only colour singlets are observed as asymptotic states.

Colour singlets:

$$\boxed{\text{mesons: } q\bar{q}, \qquad \text{baryons: } qqq.}$$

The weak interaction: start from beta decay

A basic weak process is

$$n \to p + e^- + \bar{\nu}_e.$$

At the quark level:

$$\boxed{d \to u + e^- + \bar{\nu}_e.}$$

This suggests a charged gauge boson that connects the two members of a doublet:

$$\boxed{\text{for leptons: } \nu_e \leftrightarrow e}$$

$$\boxed{\text{for quarks: } u \leftrightarrow d.}$$

The smallest non-abelian group with doublets is $SU(2)$.

Weak doublets

For one generation we write

$$\boxed{L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L.}$$

The weak charged gauge bosons change the upper component into the lower component and vice versa.

Right-handed fields are not placed in $SU(2)_L$ doublets:

$$\boxed{e_R, \quad u_R, \quad d_R \quad \text{are } SU(2)_L \text{ singlets.}}$$

Crucial word

The subscript L in $SU(2)_L$ means that the weak $SU(2)$ acts on left-handed fermions. (A parenthesis here)

Parity violation and L/R

$\boxed{\text{Weak interactions violate parity symmetry } \mathbf{x} \to -\mathbf{x}.}$

For fermion fields parity act as (why?)

$$\psi'(-\mathbf{x}, t) = \gamma^0 \psi(\mathbf{x}, t).$$

$\bar{\psi}\psi$ is parity invariant but $\bar{\psi}\gamma^5\psi$ (where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$) is not

$$\bar{\psi}'\gamma^5\psi' \to -\bar{\psi}\gamma^5\psi.$$

Define the projectors $P_{L,R}$ (remember: $\{\gamma^\mu, \gamma^5\} = 0, \ (\gamma^5)^2 = 1$)

$$\boxed{\psi_L = P_L\psi \ , \quad P_L = \frac{1}{2}(1 - \gamma^5) \ ,}$$

$$\boxed{\psi_R = P_R\psi \ , \quad P_R = \frac{1}{2}(1 + \gamma^5) \ .}$$

Then massless ψ_L and ψ_R are independent solutions to the Dirac field equation!

Chiral Fermions

Under Parity $\psi_L \leftrightarrow \psi_R$.

Weyl fermions ψ_L and ψ_R can have their own life in the Lagrangian

$$\mathcal{L} = i\bar{\psi}_L \partial\!\!\!/ \psi_L + i\bar{\psi}_R \partial\!\!\!/ \psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \ .$$

If ψ_L and ψ_R transform under different reps of the gauge group then the mass term is not allowed. This is what happens in the SM.

We can even write a mass term for a Weyl fermion (Majorana mass term)

$$\mathcal{L}_{mass} = -\frac{1}{2}\, m\, \psi_L^T\, C\psi_L + \mathrm{H.c}$$

This makes sense only for neutral fermions (e.g neutrinos).

The $SU(2)$ covariant derivative

The $SU(2)$ generators in the doublet representation are

$$T^i = \frac{\sigma^i}{2}.$$

The covariant derivative is

$$\boxed{D_\mu = \partial_\mu + ig\frac{\sigma^i}{2}W_\mu^i.}$$

In matrix form,

$$D_\mu = \partial_\mu + \frac{ig}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}.$$

Define

$$\boxed{W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2).}$$

Charged-current interactions

Using

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2),$$

we obtain charged-current interactions such as

$$\boxed{\mathcal{L}_{\mathrm{CC}} \supset -\frac{g}{\sqrt{2}} \left(\bar{\nu}_L \gamma^\mu e_L W_\mu^+ + \bar{e}_L \gamma^\mu \nu_L W_\mu^- \right).}$$

For quarks:

$$\boxed{\mathcal{L}_{\mathrm{CC}} \supset -\frac{g}{\sqrt{2}} \left(\bar{u}_L \gamma^\mu d_L W_\mu^+ + \bar{d}_L \gamma^\mu u_L W_\mu^- \right).}$$

This is the gauge-theory origin of beta decay.

The neutral W^3 problem

The same $SU(2)$ theory gives a neutral interaction through W_μ^3:

$$\mathcal{L} \supset -\frac{g}{2} W_\mu^3 \left(\bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma^\mu e_L\right)$$

for leptons.

This looks somewhat like a Z-boson interaction.

But

The observed neutral weak boson couples also to right-handed charged fermions. Pure $SU(2)$ is not enough.

We need $SU(2)_L \times U(1)_Y$.

Weak interactions are left-handed

The empirical fact is implemented by declaring:

$$\boxed{\text{Only left-handed quarks and leptons transform as } SU(2)_L \text{ doublets.}}$$

For one generation:

$$\boxed{L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L,}$$

$$\boxed{e_R, \quad u_R, \quad d_R \text{ are } SU(2)_L \text{ singlets.}}$$

The charged-current vertex contains

$$\boxed{\gamma^\mu P_L.}$$

The mass problem already appears

A Dirac mass term couples left and right:

$$\mathcal{L}_m = -m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L).$$

But if ψ_L and ψ_R transform differently under $SU(2)_L$, then this term is not gauge invariant.

$$\boxed{\text{Chiral weak interactions forbid explicit fermion masses.}}$$

Important

We postpone the resolution to Lecture 4: the Higgs mechanism and Yukawa couplings.

Why add $U(1)_Y$?

Pure $SU(2)_L$ gives $W^\pm$ and W^3.

But Nature has two neutral bosons:

$$\boxed{\gamma \quad \text{and} \quad Z.}$$

Both couple to left- and right-handed charged fermions.

So introduce an additional gauge boson B_μ of a new $U(1)_Y$:

$$\boxed{SU(2)_L \times U(1)_Y.}$$

The charge under $U(1)_Y$ is called weak hypercharge Y.

Electroweak covariant derivative

The electroweak covariant derivative is

$$\boxed{D_\mu = \partial_\mu + ig\frac{\sigma^i}{2}W_\mu^i + ig'YB_\mu.}$$

For right-handed singlets, the $SU(2)_L$ part is absent:

$$D_\mu\psi_R = (\partial_\mu + ig'Y_f B_\mu)\psi_R.$$

Notation

g is the $SU(2)_L$ coupling; g' is the hypercharge coupling.

Neutral-boson mixing: before masses

We introduce two linear combinations:

$$\boxed{W_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu,}$$

$$\boxed{B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu.}$$

Equivalently,

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}.$$

The angle θ_W is the weak mixing angle.

Recovering QED fixes hypercharge

The photon must couple as

$$\mathcal{L}_{\mathrm{em}} = -eA_\mu \bar{f}\gamma^\mu Q_f f\,.$$

From the electroweak covariant derivative, the photon coupling is consistent if

$$\boxed{g\sin\theta_W = g'\cos\theta_W = e}$$

and

$$\boxed{Q = T_3 + Y\,.}$$

Central relation

Electric charge is the unbroken combination of weak isospin and hypercharge.

Hypercharge assignments

Using

$$Q = T_3 + Y,$$

one obtains for one generation:

Field	$SU(2)_L$	Y
$Q_L = (u, d)_L^T$	doublet	$1/6$
u_R	singlet	$2/3$
d_R	singlet	$-1/3$
$L_L = (\nu, e)_L^T$	doublet	$-1/2$
e_R	singlet	-1

A hypothetical ν_R would have

$$Y = 0, \qquad Q = 0,$$

so it would be sterile with respect to SM gauge interactions.

Electromagnetic and neutral currents

The neutral-current Lagrangian separates as

$$\boxed{\mathcal{L}_{\mathrm{NC}} = \mathcal{L}_{\mathrm{QED}} + \mathcal{L}_{\mathrm{NC}}^{Z}.}$$

The photon part is

$$\boxed{\mathcal{L}_{\mathrm{QED}} = -eA_\mu \sum_f Q_f \bar{f}\gamma^\mu f.}$$

The Z-boson coupling can be written as

$$\boxed{\mathcal{L}_{\mathrm{NC}}^{Z} = -\frac{g}{\cos\theta_W} Z_\mu \bar{f}\gamma^\mu \left(T_3 P_L - Q\sin^2\theta_W\right) f.}$$

Vector and axial couplings

It is often useful to write

$$\mathcal{L}^{Z}_{\mathrm{NC}} = -\frac{g}{2\cos\theta_W} Z_\mu \bar{f}\gamma^\mu \left(g^f_V - g^f_A\gamma^5\right) f.$$

Then

$$\boxed{g^f_A = T^f_3, \qquad g^f_V = T^f_3 - 2Q_f \sin^2\theta_W.}$$

Examples:

$$\nu_L: \quad Q = 0, \quad T_3 = +\frac{1}{2},$$

$$e_L: \quad Q = -1, \quad T_3 = -\frac{1}{2}, \qquad e_R: \quad T_3 = 0.$$

Charged currents in the electroweak theory

The charged-current interaction is

$$\boxed{\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu} \right).}$$

For one generation,

$$J_\mu^+ = \bar{\nu}_L \gamma_\mu e_L + \bar{u}_L \gamma_\mu d_L.$$

At this stage we ignore quark mixing.

Later refinement

With three generations, d_L is replaced by CKM mixtures in the quark charged current.

What has been unified?

Before electroweak theory:

electromagnetism and weak interactions look completely different.

Electroweak theory says they arise from one gauge structure:

$$\boxed{SU(2)_L \times U(1)_Y.}$$

The photon and Z are mixtures of W^3 and B:

$$\boxed{\begin{aligned} A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu, \\ Z_\mu &= \cos\theta_W W^3_\mu - \sin\theta_W B_\mu. \end{aligned}}$$

But we have not yet explained why W and Z are massive.

Two elephants in the room

The electroweak gauge theory has two major unresolved issues:

1. Gauge symmetry forbids explicit gauge-boson masses:

$$M_W,\ M_Z \neq 0 \quad \text{experimentally.}$$

2. Chiral gauge representations forbid explicit fermion masses:

$$-m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

is not generally gauge invariant.

$\boxed{\text{Both problems will be solved by the Higgs mechanism.}}$

Summary of Lecture 3

$$\boxed{\text{QED is a local } U(1) \text{ gauge theory.}}$$

$$\boxed{\text{Non-abelian gauge theories have } [T^a, T^b] \neq 0.}$$

$$\boxed{F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu.}$$

$$\boxed{SU(3)_c \text{ gives QCD with eight gluons and gluon self-interactions.}}$$

$$\boxed{SU(2)_L \times U(1)_Y \text{ gives the electroweak gauge structure.}}$$

One-sentence message

$\boxed{\text{The forces of the Standard Model are dictated by local gauge symmetry.}}$

Lecture 4 begins with the unavoidable question:

$\boxed{\text{If gauge symmetry forbids masses, how does Nature give masses to } W^{\pm}, Z \text{ a}}$

Answer:

$\boxed{\text{spontaneous symmetry breaking and the Higgs mechanism.}}$

Tutorial problems

1. Show that local $U(1)$ invariance requires a covariant derivative.
2. Derive the QED interaction from $\bar{\psi} i \not{D} \psi$.
3. Count the gluons of $SU(3)_c$ and identify why they self-interact.
4. Use $Q = T_3 + Y$ to derive SM hypercharges for one generation.
5. Compute g_V^f and g_A^f for ν, e, u, d.

Reading for this lecture

- D. Griffiths, Introduction to Elementary Particle Physics, 2008.
- M. Peskin, Concepts of Elementary Particle Physics, Oxford Master Series, 2019.
- B. Gripaios, From QM to the SM, arXiv:2005.06355
- For learning Group Theory I suggest the book by H. Georgi, *Lie Algebras in Particle Physics*, 2nd ed. 1999