Lecture 2: Relativistic Quantum Fields

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Introductory Lectures on the Standard Model

Purpose of Lecture 2

Central question

How does one pass from relativistic wave equations to a quantum theory with particles, antiparticles, spin, statistics, and interactions?

$\boxed{\text{fields become operators, and particles become excitations of fields.}}$

Roadmap of the lecture

1. Classical field theory and Noether currents
2. Quantizing the real scalar field
3. Complex scalar fields and antiparticles
4. Quantizing the Dirac field and Pauli exclusion
5. Gauge field quantization and photon polarizations
6. Interactions, Dyson expansion, and amplitudes
7. Pair production and Compton scattering

$$\boxed{\mathcal{L} \to \text{fields} \to a, a^\dagger \to \text{states} \to \mathcal{M} \to \Gamma, \sigma}$$

From Theoretical Physics to Experimental Physics

Fields and action

A classical field is a function on spacetime:

$$\phi = \phi(x^\mu).$$

The action is written using a Lagrangian density:

$$\boxed{S = \int \mathrm{d}^4 x\, \mathcal{L}(\phi, \partial_\mu \phi).}$$

Important assumption:

$\boxed{\text{locality: the Lagrangian couples fields at the same spacetime point.}}$

From now on, we usually call $\mathcal{L}$ simply "the Lagrangian".

Euler–Lagrange equation for fields

Vary the action with $\phi \to \phi + \delta\phi$:

$$\delta S = \int \mathrm{d}^4 x \left[\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta(\partial_\mu \phi) \right].$$

After integrating by parts:

$$\delta S = \int \mathrm{d}^4 x \left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \right] \delta\phi.$$

Thus

$$\boxed{\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0.}$$

Example: Klein–Gordon Lagrangian

For a real scalar field:

$$\boxed{\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2\ .}$$

Apply the Euler–Lagrange equation:

$$\frac{\partial\mathcal{L}}{\partial\phi} = -m^2\phi, \qquad \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = \partial^\mu\phi.$$

Therefore

$$\boxed{(\partial_\mu\partial^\mu + m^2)\phi = 0.}$$

The Klein–Gordon equation is now a **field equation**, not a one-particle probabilistic wave equation.

Noether's theorem

If the action is invariant under

$$\phi \to \phi + \delta\phi,$$

then the Lagrangian can change at most by a total derivative:

$$\delta\mathcal{L} = \partial_\mu K^\mu.$$

On the equations of motion one obtains a conserved current:

$$\boxed{\partial_\mu J^\mu = 0, \qquad J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi - K^\mu.}$$

$\boxed{\text{Noether's Theorem: symmetry} \Longrightarrow \text{conserved current.}}$

Example: complex scalar current

For a complex Klein–Gordon field:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi.$$

It is invariant under a global phase rotation:

$$\phi \to e^{i\alpha} \phi.$$

For small real α:

$$\delta\phi = i\alpha\phi, \qquad \delta\phi^* = -i\alpha\phi^*.$$

The conserved current is

$$\boxed{J^\mu = i\left(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*\right).}$$

This is precisely the current encountered in relativistic quantum mechanics.

Energy-momentum tensor

Spacetime translations

$$x^\mu \to x^\mu + a^\mu$$

lead to conserved energy and momentum.
The corresponding Noether current is the energy-momentum tensor:

$$\boxed{T^\mu_{\ \nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_{\ \nu} \mathcal{L}.}$$

- $\partial_\mu T^\mu_{\ 0} = 0$: energy conservation.
- $\partial_\mu T^\mu_{\ i} = 0$: momentum conservation.

Poincare symmetry controls energy, momentum, and angular momentum.

Symmetry almost fixes the Lagrangian

For electromagnetism, require:

- Lorentz invariance;
- gauge invariance $A_\mu \to A_\mu + \partial_\mu \Lambda$.

The gauge-invariant object is

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.}$$

The lowest-derivative Lagrangian is

$$\boxed{\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}.}$$

Local phase symmetry of matter fields leads to

$$\boxed{\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu.}$$

Canonical quantization of a real scalar field

Start with

$$\boxed{\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2\ .}$$

The conjugate momentum is

$$\pi(x) = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \dot{\phi}(x).$$

Quantize by imposing equal-time commutators:

$$\boxed{[\phi(\boldsymbol{x},t),\pi(\boldsymbol{y},t)] = i\delta^3(\boldsymbol{x}-\boldsymbol{y}),}$$

$$[\phi(\boldsymbol{x},t),\phi(\boldsymbol{y},t)] = [\pi(\boldsymbol{x},t),\pi(\boldsymbol{y},t)] = 0.$$

$\boxed{\text{we quantize the field, not the particle position.}}$

Mode expansion

The real scalar field is expanded as

$$\boxed{\phi(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}} \left(a_{\boldsymbol{p}} e^{-ip\cdot x} + a^\dagger_{\boldsymbol{p}} e^{ip\cdot x} \right),}$$

where

$$E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2 + m^2}.$$

Reality of ϕ means Hermiticity:

$$\phi^\dagger = \phi.$$

The equal-time commutators become

$$\boxed{[a_{\boldsymbol{p}}, a^\dagger_{\boldsymbol{q}}] = (2\pi)^3 2E_{\boldsymbol{p}} \delta^3(\boldsymbol{p} - \boldsymbol{q}),}$$

$$[a_{\boldsymbol{p}}, a_{\boldsymbol{q}}] = [a^\dagger_{\boldsymbol{p}}, a^\dagger_{\boldsymbol{q}}] = 0.$$

The field is a collection of oscillators

The Hamiltonian is

$$H = \frac{1}{2} \int \mathrm{d}^3 x \left(\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 \right) .$$

After inserting the mode expansion:

$$H = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}} \frac{E_{\boldsymbol{p}}}{2} \left(a_{\boldsymbol{p}} a_{\boldsymbol{p}}^\dagger + a_{\boldsymbol{p}}^\dagger a_{\boldsymbol{p}} \right) .$$

This is a continuum of harmonic oscillators, one for each $\boldsymbol{p}$.

$$\boxed{\text{field quantization} = \text{quantizing infinitely many oscillators.}}$$

Particles as excitations

Define the vacuum by

$$\boxed{a_{\boldsymbol{p}}|0\rangle = 0 \qquad \forall \boldsymbol{p}.}$$

A one-particle state is

$$\boxed{|\boldsymbol{p}\rangle = a^\dagger_{\boldsymbol{p}}|0\rangle.}$$

A two-particle state is

$$|\boldsymbol{p}, \boldsymbol{q}\rangle = a^\dagger_{\boldsymbol{p}} a^\dagger_{\boldsymbol{q}}|0\rangle.$$

Since

$$[a^\dagger_{\boldsymbol{p}}, a^\dagger_{\boldsymbol{q}}] = 0,$$

we have

$$|\boldsymbol{p}, \boldsymbol{q}\rangle = |\boldsymbol{q}, \boldsymbol{p}\rangle.$$

spin-0 field quanta obey Bose–Einstein statistics.

What do the fields do to the particles?

The real field $\phi(x)$ destroys a particle of momentum p at the space-time point x:

$$\boxed{\langle 0|\, \phi(x)\, |p\rangle = e^{-ip\cdot x}}$$

or creates a particle with momentum p at x

$$\boxed{\langle p|\, \phi(x)\, |0\rangle = e^{ip\cdot x}}$$

There is a pion field, there is a Higgs field, there is an electron field, etc. For example here the pion field $\phi(x)$ destroys the pion particle π with momentum p at x

$$\langle 0|\, \text{pion field } \phi(x)\, |\pi(p)\rangle = e^{-ip\cdot x}$$

Everywhere, the energy E is positive definite!

Vacuum energy and normal ordering

The vacuum energy contains

$$\langle 0|H|0\rangle = \int \mathrm{d}^3 p \, \frac{E_{\boldsymbol{p}}}{2} \times \delta^3(0),$$

which is divergent. This is our first infinity, and it reappears in gravity as the cosmological constant problem.

Ignoring gravity, only energy differences are measured. We define normal ordering:

$$: a_{\boldsymbol{p}} a_{\boldsymbol{p}}^\dagger := a_{\boldsymbol{p}}^\dagger a_{\boldsymbol{p}}.$$

The normally ordered Hamiltonian is

$$\boxed{: H := \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}} \, E_{\boldsymbol{p}} \, a_{\boldsymbol{p}}^\dagger a_{\boldsymbol{p}}.}$$

Complex scalar field and antiparticles

For a complex scalar field $\phi(x)$

$$\boxed{\phi(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}} \left(a_{\boldsymbol{p}} e^{-ipx} + b^\dagger_{\boldsymbol{p}} e^{ipx} \right).}$$

There are two independent creation operators:

$$\boxed{a^\dagger_{\boldsymbol{p}} : \text{ particle}, \qquad b^\dagger_{\boldsymbol{p}} : \text{ antiparticle.}}$$

The Hamiltonian is

$$: H := \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}} E_{\boldsymbol{p}} \left(a^\dagger_{\boldsymbol{p}} a_{\boldsymbol{p}} + b^\dagger_{\boldsymbol{p}} b_{\boldsymbol{p}} \right).$$

$\boxed{\text{complex fields naturally contain particles and antiparticles.}}$

Conserved charge

The complex scalar theory has a global $U(1)$ symmetry:

$$\phi \to e^{i\alpha}\phi.$$

The conserved charge becomes

$$\boxed{Q = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}} \left(a_{\boldsymbol{p}}^\dagger a_{\boldsymbol{p}} - b_{\boldsymbol{p}}^\dagger b_{\boldsymbol{p}} \right).}$$

Thus Q counts

$$\boxed{\#\text{particles} - \#\text{antiparticles.}}$$

Antiparticles have the same mass and opposite charge.

Dirac field quantization

The free Dirac Lagrangian **for fields** is

$$\boxed{\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi.}$$

The field (Fourier!) expansion is

$$\boxed{\psi(x) = \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E} \left(c^s_{\boldsymbol{p}} u^s(p) e^{-ipx} + d^{s\dagger}_{\boldsymbol{p}} v^s(p) e^{ipx} \right).}$$

Interpretation:

$$\boxed{c^{s\dagger}_{\boldsymbol{p}} : \text{ fermion}, \qquad d^{s\dagger}_{\boldsymbol{p}} : \text{ antifermion.}}$$

The label $s = 1, 2$ denotes the two spin states.

Why anticommutators?

If one used commutators for the Dirac field, antiparticles would contribute with the wrong sign to the energy.
The consistent choice is

$$\boxed{\{c_{\boldsymbol{p}}^r, c_{\boldsymbol{q}}^{s\dagger}\} = (2\pi)^3 2E\delta^{rs}\delta^3(\boldsymbol{p}-\boldsymbol{q})\ , \quad \{c_{\boldsymbol{p}}^r, c_{\boldsymbol{q}}^{s}\} = 0\ , \quad \{c_{\boldsymbol{p}}^{r\dagger}, c_{\boldsymbol{q}}^{s\dagger}\} = 0\ .}$$

$$\boxed{\{d_{\boldsymbol{p}}^r, d_{\boldsymbol{q}}^{s\dagger}\} = (2\pi)^3 2E\delta^{rs}\delta^3(\boldsymbol{p}-\boldsymbol{q})\ , \quad \{d_{\boldsymbol{p}}^r, d_{\boldsymbol{q}}^{s}\} = 0\ , \quad \{d_{\boldsymbol{p}}^{r\dagger}, d_{\boldsymbol{q}}^{s\dagger}\} = 0\ .}$$

Then charge counts particles minus antiparticles, and both contribute positively to the energy.

$\boxed{\text{spin-half fields must be quantized with anticommutators.}}$

Pauli exclusion from QFT !

Because of anticommutation,

$$\{c_{\boldsymbol{p}}^{r\dagger}, c_{\boldsymbol{q}}^{s\dagger}\} = 0$$

we have

$$|\boldsymbol{p}, r; \boldsymbol{q}, s\rangle = c_{\boldsymbol{p}}^{r\dagger}\, c_{\boldsymbol{q}}^{s\dagger}\, |0, 0\rangle = -c_{\boldsymbol{q}}^{s\dagger}\, c_{\boldsymbol{p}}^{q\dagger}\, |0, 0\rangle = -|\boldsymbol{q}, s; \boldsymbol{p}, r\rangle.$$

Therefore identical fermions, say electrons,

$$|e^-(p, s), e^-(p, s)\rangle = 0\ ,$$

cannot occupy the same quantum state.

$\boxed{\text{Pauli exclusion follows from consistent quantization of spin-half fields.}}$

This is one of the conceptual payoffs of Lecture 2.

Why gauge fields are subtle

For electromagnetism, A_0 has no time derivative in the Lagrangian. Thus its conjugate momentum vanishes:

$$\pi^0 = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0.$$

The Hamiltonian contains A_0 as a Lagrange multiplier enforcing Gauss' law:

$$\nabla \cdot \boldsymbol{E} = 0$$

for free electromagnetism.

$\boxed{\text{gauge theories are constrained systems.}}$

Fix the gauge completely.

Coulomb gauge and photon polarizations

Choose Coulomb gauge:

$$\partial_\mu A^\mu = 0, \qquad A^0 = 0.$$

A plane wave has

$$A^i = \epsilon^i e^{-ip\cdot x}, \qquad p^2 = 0.$$

The gauge condition implies

$$\boxed{\boldsymbol{\epsilon} \cdot \boldsymbol{p} = 0.}$$

Thus the photon has two physical polarizations.
For waves moving in the z-direction, circular polarizations may be chosen

$$\boxed{\epsilon_\mu^{L,R} = \frac{1}{\sqrt{2}}(0, -1, \pm i, 0).}$$

Quantizing the photon field

The spatial components ($i = 1, 2, 3$) are expanded as

$$\boxed{A_i(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E} \sum_P \left(a_{\boldsymbol{p}}^P \epsilon_i^P e^{-ip\cdot x} + a_{\boldsymbol{p}}^{P\dagger} \epsilon_i^{P*} e^{ip\cdot x} \right).}$$

The polarization completeness relation is

$$\boxed{\sum_P \epsilon_i^P \epsilon_j^P = \delta_{ij} - \frac{p_i p_j}{\boldsymbol{p}^2}.}$$

The Hamiltonian is

$$\boxed{H = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E} \sum_P E\, a_{\boldsymbol{p}}^{P\dagger} a_{\boldsymbol{p}}^P.}$$

Free theories are dull

So far all Lagrangians were quadratic in fields.
They lead to linear equations and free particles.

Interacting theories contain terms with more than two fields:

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}.}$$

Examples:

$$\lambda\phi^4, \qquad -eA_\mu\bar{\psi}\gamma^\mu\psi.$$

$\boxed{\text{interactions allow particles to appear, disappear, and transform.}}$

Interaction picture

Split the Hamiltonian:

$$H = H_0 + H_1.$$

In the **interaction picture**:

- operators evolve with H_0;
- states evolve with H_I.

The time evolution operator is

$$\boxed{U(t, t_0) = T \exp\left[-i \int_{t_0}^{t} \mathrm{d}t'\, H_I(t')\right].}$$

Here T denotes time ordering.[1]

$\boxed{\text{time ordering enforces causal ordering in perturbation theory.}}$

[1]Time ordering:

$$T\{H(t_1)H(t_2)\} = \Theta(t_1 - t_2)H(t_1)H(t_2) + \Theta(t_2 - t_1)H(t_2)H(t_1)\ .$$

Perturbative expansion

Expand the Dyson operator:

$$U(t, t_0) = 1 - i \int_{t_0}^{t} \mathrm{d}t_1 \, H_I(t_1) + \frac{(-i)^2}{2!} \int_{t_0}^{t} \mathrm{d}t_1 \int_{t_0}^{t} \mathrm{d}t_2 \, T\{H_I(t_1) H_I(t_2)\} + \cdots .$$

In particle physics we use asymptotic initial and final states S-matrix:

$$\boxed{S_{fi} = \langle f | \, U(+\infty, -\infty) \, | i \rangle .}$$

This defines the scattering amplitude.

Feynman diagrams are mnemonics for this perturbation series.

The invariant matrix element

Momentum conservation always gives a delta function:

$$\boxed{S_{fi} = \langle f|\, U(+\infty, -\infty)\,|i\rangle = i(2\pi)^4\delta^4(p_f - p_i)\,\mathcal{M}.}$$

The physics is in the matrix element $\mathcal{M}$.

$$\boxed{\mathcal{M} \to |\mathcal{M}|^2 \to \Gamma, \sigma.}$$

This connects Lecture 2 to the transition-rate formulae introduced at the end of Lecture 1.

Pair production: $\gamma \to e^+e^-$

The electromagnetic interaction Hamiltonian density is

$$\mathcal{H}_I = +eA_\mu \bar{\psi}\gamma^\mu \psi.$$

At leading order,

$$-i \int \mathrm{d}^4x \, \mathcal{H}_I$$

connects an incoming photon to an outgoing electron-positron pair.
The matrix element is

$$\boxed{i\mathcal{M} = -ie\,\epsilon_\mu^P \, \bar{u}^{s_1}(p_1)\gamma^\mu v^{s_2}(p_2).}$$

Interpretation:

- ϵ_μ^P: incoming photon;
- $\bar{u}$: outgoing electron;
- v: outgoing positron;
- $-ie\gamma^\mu$: QED vertex.

Feynman-rule logic from pair production

The calculation suggests the first QED rule:

$$\boxed{\text{vertex factor:} \ \ -ie\gamma^{\mu}.}$$

External lines give wavefunctions:

$$\boxed{\gamma : \epsilon_{\mu} \text{ or } \epsilon_{\mu}^{*} \qquad e^{-} : u \text{ or } \bar{u}, \qquad e^{+} : v \text{ or } \bar{v}.}$$

Momentum conservation is supplied by

$$(2\pi)^4\delta^4(p_f - p_i).$$

Pedagogical point

Feynman rules are not magic: they summarize the operator algebra of QFT.

Wick's theorem and propagators

To compute second-order processes, we need time-ordered products.
Wick's theorem says schematically

$$\boxed{T\{\phi(x_1)\phi(x_2)\cdots\} =: \phi(x_1)\phi(x_2)\cdots : + \text{contractions.}}$$

For a scalar field:

$$\boxed{\Delta_F(x-y) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)}.}$$

For a Dirac field:

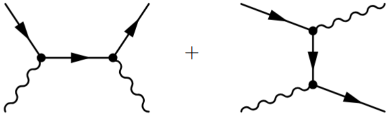
$$\boxed{S_F(x-y) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{i}{\not{p} - m + i\epsilon} e^{-ip\cdot(x-y)}.}$$

Compton scattering

Consider

$$e^- + \gamma \to e^- + \gamma.$$

It first appears at second order in the QED interaction.
There are two contractions, hence two diagrams:



$\boxed{\text{the propagator represents the internal virtual electron.}}$

Compton amplitude

With incoming electron momentum p, incoming photon k, outgoing photon k':

$$\boxed{\begin{aligned} i\mathcal{M} = (-ie)^2 \epsilon^{*\prime}_{\mu} \bar{u}' \Bigg[& \gamma^{\mu} \frac{i(\not{p} + \not{k} + m)}{(p+k)^2 - m^2} \gamma^{\nu} \\ & + \gamma^{\nu} \frac{i(\not{p} - \not{k}' + m)}{(p-k')^2 - m^2} \gamma^{\mu} \Bigg] u \, \epsilon_{\nu}. \end{aligned}}$$

The two terms interfere in $|\mathcal{M}|^2$.

$\boxed{\text{real observables require squaring and integrating over phase space.}}$

Summary of Lecture 2

$\boxed{\text{The Lagrangian gives equations of motion and symmetries.}}$

$\boxed{\text{Quantized fields contain creation and annihilation operators.}}$

$\boxed{\text{Particles are excitations of fields.}}$

$\boxed{\text{Complex fields contain antiparticles.}}$

$\boxed{\text{Spin-half fields require anticommutation relations.}}$

$\boxed{\text{Feynman diagrams summarize perturbative QFT amplitudes.}}$

Conceptual arc of Lecture 2

Classical fields $\rightarrow$ canonical quantization $\rightarrow$ particles

complex fields $\rightarrow$ antiparticles and charge

Dirac fields $\rightarrow$ spin, anticommutators, Pauli principle

gauge fields $\rightarrow$ two photon polarizations

interactions $\rightarrow$ amplitudes and Feynman diagrams

Next: gauge field theories and the structure of the Standard Model.

Suggested exercises

1. Derive the Euler–Lagrange equation for the KG Lagrangian.
2. Derive the Noether current for $\phi \to e^{i\alpha}\phi$.
3. Show that the real scalar Hamiltonian is a sum of harmonic oscillators.
4. Show the creation of the electron from the vacuum:

$$\langle e^-(p,s)|\,\bar{\Psi}(x)\,|0\rangle = \bar{u}_s(p)\,e^{+ip\cdot x}$$

5. Starting from the QED vertex, reconstruct the amplitude for $e^+ + e^- \to \gamma + \gamma$.

Reading for this lecture

- F. Mandl and G. Shaw, Quantum Field Theory, Wiley, 2nd ed., 2010.
- M. Peskin and D. Schroeder, Introduction to Quantum Field Theory, Addison-Wesley, 1995.
- A. Zee, QFT in a Nutshell, Princeton Univ. Press, 2nd ed., 2010.
- B. Gripaios, From QM to the SM, arXiv:2005.06355

Back-Up: Number operator and negative-frequency modes

The particle-number operator is

$$\boxed{N = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E}\, a^\dagger_{\boldsymbol{p}} a_{\boldsymbol{p}}.}$$

For a real scalar field, N is not generally conserved.

The negative-frequency part of the field is no longer a negative-energy particle:

$$\phi \sim a e^{-ipx} + a^\dagger e^{ipx}.$$

$\boxed{\text{negative frequency accompanies creation or annihilation operators.}}$

This is how QFT resolves the negative-energy problem.